VISUALISING FRACTIONS

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Abstract

Learning Mathematics is something difficult for many students, as they cannot visualise the abstract concepts. Fraction is one of them. Student cannot visualise the process of operations on fractions. The algorithms of those processes are too complicated for them. By modelling the fractions (fullness) and its operations with Geometer Sketchpad (GSP) and medic box (container), the same operations on fractions can be made easier and provide a better understanding to students. The main idea of this paper is try to show the transformation of abstract fractions into countable natural numbers so that can be used in operations.

Introduction

The main objective of this paper is to help students get a better understanding in fractions, both the concept and the operations on fractions. A fraction is one of the most difficult topics in primary mathematics. "Difficulty with fractions (including decimals and percentage) is pervasive and is a major obstacle to further progress in mathematics, including algebra" (National Mathematics Advisory Panel, 2008). Natural numbers are discrete and countable for students. It is difficult for students to visualise the concept of fractions as a ratio of two numbers since it is an abstract concept. There are so many fractions can be found on a number line between any two natural numbers. This makes fractions uncountable for primary students, as they cannot place them on the number line as they do with natural numbers. This also make operations on fractions are much more complicated compared to operations on natural numbers. They cannot "count on" for fractions to do additions as they do count on for the natural numbers. There are too many conditions and rules for the students to follow while performing the operations on fractions. Most of them are very much different from operations on natural numbers. The main idea in this paper is try to transform fractions into the form of natural numbers so that operations can be easily performed.

Difficulty in Learning Fractions

Students have to master two main skills to learn fraction. The first skill is related to the concept of fractions such as recognise-read-write fractions, convert proper-improper-mixed number and compare fractions. The second skill is the knowledge of applying the four basic operations on fractions. Table 1 shows the summary of the skills and the difficulties involved.

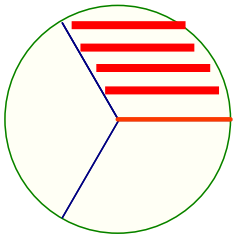
Table 1
Skills, Difficulties and Normal Strategies

Skills	Difficulties	Normal Strategies
recognise-read-write fractions	At least 3 different concepts: part-whole, part of collection, ratio.	Using real materials such as: cake or pie
convert improper-mixed number, compare fractions	operations: multiply with suitable number	using semi-concrete materials such as fractions circle or fractions bar
operations on fractions	different operations involve different procedures	standard algorithm, with key point: "Least Common Multiple for denominator"

Representing Mathematical Idea

The using of supporting material is usually parallel to Bruner modes of representing mathematical idea such from enactive (concrete objects) to iconic (pictures or drawing) to symbolic (abstract symbols) shown in Table 2.

Table 2
Bruner Modes of Representing Mathematical Idea

Enactive	Iconic	Symbolic
Fractions as Part of a Set	Fractions as Part of a Whole	Fractions as Ratio of Two Numbers
		$\frac{1}{3}$

It is difficult to represent the operations on fractions in such strategies. The operations usually are through standard algorithm as shown in Table 3.

Table 3
Standard Algorithm for Operations on Fractions

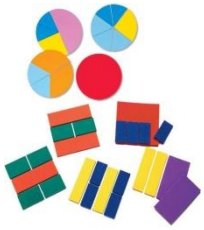
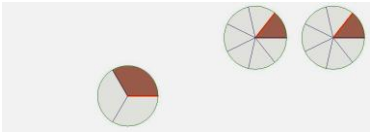
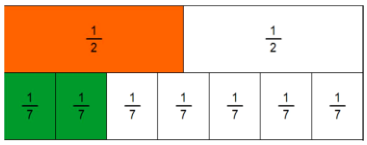
Addition / Subtraction	Multiplication	Division
$\frac{1}{3}+\frac{2}{7}=\frac{1x7}{3x7}+\frac{2x3}{7x3}$ $=\frac{7}{21}+\frac{6}{21}=\frac{7+6}{21}=\frac{13}{21}$	$\frac{1}{3}x\frac{2}{7}=\frac{1x2}{3x7}=\frac{2}{21}$	$\frac{1}{3}\div\frac{2}{7}=\frac{1x7}{3x2}=\frac{7}{6}$

Inconsistency in Using Supportive Learning Materials

The change of strategies or inconsistency in using supportive materials may confuse students. The complicated procedures in standard algorithm also make learning fraction more difficult for many students.

Table 4

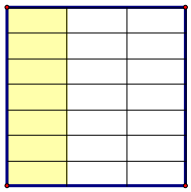
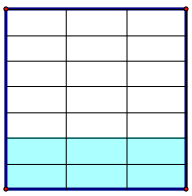
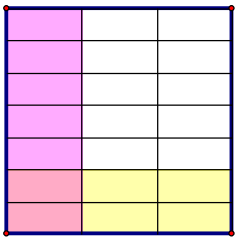
The Change of Strategies in Learning Fractions

Concrete	Semi Concrete	Symbolic
	 	$\frac{1}{3}+\frac{2}{7}=\frac{1x7}{3x7}+\frac{2x3}{7x3}$ $=\frac{7}{21}+\frac{6}{21}=\frac{7+6}{21}=\frac{13}{21}$

The 'Fractions Circles' and 'Fraction Bars' are not suitable to use for visualising operation on fractions. "Visualising Fractions with Rectangle" (2nd National Conference on Graphing Calculator and GSP, 2008, Benny Kong) which refers to the strategy of representing fractions by using rectangles has been found to be more effective.

Table 5

The Representation of fraction by Rectangles

<table>
<tr><td>$\frac{1}{3}$</td><td>$\frac{2}{7}$</td><td colspan="2">Operations</td></tr>
<tr><td rowspan="3">  </td><td rowspan="3">  </td><td>compare</td><td>7 > 6</td></tr>
<tr><td>add</td><td>7+6 = 13 of 21 pcs</td></tr>
<tr><td>subtract</td><td>7+6 = 13 of 21 pcs</td></tr>
<tr><td colspan="2">Rectangles With 7 Rows & 3 Columns = 21 pcs</td><td>divide</td><td>7 / 6</td></tr>
<tr><td>1 column of 3 columns
shows 7 pcs</td><td>2 rows of 7 rows
shows 6 pcs</td><td>multiply</td><td> 
2 (overlap) of 21pcs</td></tr>
</table>

Alternative Strategy in Learning Fractions by Transforming into Whole Number

Learning must be fun and meaningful for the students. Fun relates to student centred and materials centred learning. Meaningful always relates to contextual learning and learning based on real life experience.

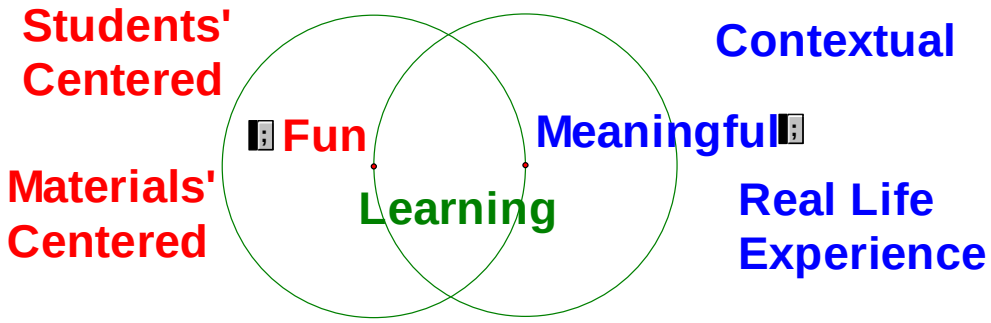


Figure1. Fun and Meaningful Learning.

These effective learning method or representation of fraction should happen only at the early stage of learning fractions where most students were exposed to real or concrete materials when fractions were first introduced. The using of concrete materials any sooner will bring changes to the process of abstract symbols at the later stage of learning fractions. Students may have to memorise mechanical steps to compare and perform operations on fractions. This is meaningless and is of no fun for many students.

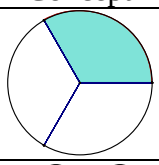
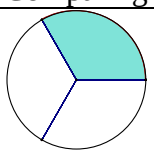
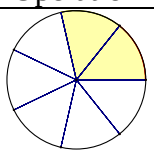
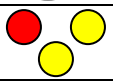
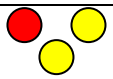
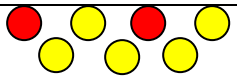
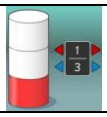
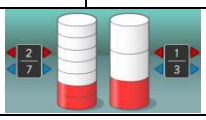
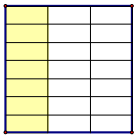
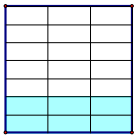
A Consistent Use of Supporting Materials

Based on the findings of a research conducted by the researcher himself on "Visualising fractions With Rectangles" which was presented at the 2nd National Conference Graphing Calculator" in 2008, forming rectangles is one of the better ways in representing fractions. Based on the feedback from the teacher trainee after completing their teaching practicum, concrete materials play an important role in helping students to understand and master fractions concept-operations. "Pocket Fraction" by Tan (KPLI, 2009) and "Egg Tray" by Siti Aminah (PISMP, 2011) both show that students can understand the concept of fraction and master the operation (addition and subtraction) on fractions.

Fractions as "Fillness" or "Fullness"

The researcher tried out a new concept by using newly developed supporting material for learning fractions. This was done by introducing another concept of fractions such as "fillness" or "fullness" which is also referred to as "**rectangular container"**. Table 5 compares the difference of using different supporting materials.

Table 5
The Representation of Fraction by using Rectangles

	Concept	Comparing	Operation
Part Of A Whole			
Part Of A Set			
Ratio	1/3	1/3	2/7
Fullness			
Fullness On Rectangular Container/Base		1 row of 3 rows	 2 columns of 7 columns
Fullness On Rectangular Container/Base (GSP)			

The "Fullness" concept can transform the operations back to normal number with the common base or reference (the capacity of the container). Table 6 shows the using of "Fullness" concept for operations on fractions.

Table 6
The "Fullness" concept for operations on fractions

	$\frac{1}{3}$	1 row of 3 rows	$\frac{2}{7}$	2 columns of 7 columns	Operation	
Addition					(7 pcs + 6 pcs) of 21 pcs	13/21
Subtraction					(7 pcs - 6 pcs) of 21 pcs	1/21
Multiplication					2 (overlap) of 21 pcs	2/21
Division					7 pcs ÷ 6 pcs (1^{st} number divide by 2^{nd} number)	7/6

Students will operate using one number (the numerator). The denominator is referred as the container which is fixed during the process of operations. The numerators are natural number. They are countable and normal operations can be easily performed.

Students' Perception

"Fractions as Fullness" concept was shared with 27 students from the '*Ijazah Sarjana Muda Perguruan'* (*PISMP)* Mathematics major students who took MTE 3109 course (Teaching Numbers, Fractions, Decimals, and Percentage). Table 7 shows their perception.

Table 7

Students' Perception on the concept of 'Fractions as Fullness'

	Strongly Disagree (1)	Disagree (2)	Agree (3)	Strongly Agree (4)
1. Do you agree Maths (Fractions) difficult to learn?	4 14.8%	6 22.2%	13 48.1%	4 14.8%
2. Do you agree Maths (Fractions) is abstract?	0	5 18.5%	12 44.4%	10 37.0%
3. Do you agree visualising can help to master Mathematics (Fractions) Concept / Operations?	0	0	17 63.0%	10 37.0%
4. Do you agree "Fractions as Fullness" can help to Visualise Maths (Fractions) Concept / Operations?	0	0	7 26.0%	20 74.0%
5. Will you try to use "Fractions as Fullness" in you teaching of Maths (Fractions) Concept / Operations?	0	0	10 37.0%	17 63.0%

10 or 37.0% of the respondents disagreed that fraction is a topic that is difficult to be learnt (14.8% strongly disagreed and 22.2% disagreed) while more than half (62.9%) agreed. Majority too agreed that fraction is an abstract topic to learn. This probably is because they are good students and most of them obtained good results in Mathematics and thus do not face difficulties in learning Mathematics.

All of them agreed that visualisation helped in learning fractions (63.0% agreed and 37.0% strongly agreed) and all of them also agreed that "Fractions as Fullness" can be used to learn fractions (26.0% agreed and 74.0% strongly agreed). All of them agreed that they tried to use "Fractions as Fullness" in teaching of fractions (37.0% agreed and 63.0% strongly agreed).

Conclusion

The PISMP Mathematics major students agreed that "Fractions as Fullness" method is effective in learning fractions. Some of them applied "Fractions as Fullness" method during their practicum.

References

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Delazer, M., & Benke, Th. (1997). Arithmetic facts without meaning. *Cortex*, 33, 697-710.